Advanced Quantitative Methods:

Generalized Methods of Moments
Asset Pricing

Ingmar Nolte

Room: C48

I.Nolte@lancaster.ac.uk

Office Hours: whenever you like

Webpage:

http://www.lancaster.ac.uk/staff/nolte/

Password: 42GARCH

Overview:

- Consumption Based Models
- Factor Models
- Estimation

Motivation:

Asset pricing theory is the attempt to understand how prices are formed. The fundamental idea is that prices are expected discounted payoffs. The challenge is to get “expected discounted” right.

- Positive approach – describes how the world *does* work
- Normative approach – describes how the world *should* work

Basic Pricing Equation:

Asset pricing can be summarized by two simple equations:

$$P_t = \mathrm{E}_t[m_{t+1} X_{t+1}], \tag{1}$$

with

$$m_{t+1} = f(\text{data, parameters}),$$

where P_t denotes the asset price, X_{t+1} the asset payoff and m_{t+1} the stochastic discount factor.

We can estimate an asset pricing model by method of moments by considering the conditional moment condition

$$P_t - \mathrm{E}_t[m_{t+1} X_{t+1}] = 0 \quad \text{or} \quad 1 - \mathrm{E}_t[m_{t+1} R_{t+1}] = 0.$$

Adding instruments Z_t we can obtain the unconditional moment conditions as

$$\mathrm{E}[(P_t - m_{t+1} X_{t+1}) Z_t] = 0 \quad \text{or} \quad \mathrm{E}[(1 - m_{t+1} R_{t+1}) Z_t] = 0.$$

Consumption-Based Model

We consider an investor who faces the decision how much to consume today and how much to invest in order to consume in the future and what portfolio of assets to hold. We consider a time-separable linear utility function $U(C_t, C_{t+1})$ such that

$$U(C_t, C_{t+1}) = u(C_t) + \beta \mathrm{E}_t[u(C_{t+1})],$$

where β is a parameter that reflects the subjective discount factor.

E.g. we can have the power utility function

$$u(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}.$$

- γ governs the curvature – risk-aversion
- Thus the investor prefers a smooth consumption stream over time and states of the world

- e_t – the original consumption (exogenous income) at a time t
- ξ – the amount of assets he chooses to buy/sell
- The investor can freely buy or sell as much of the payoff X_{t+1} as he wishes, at time t at a price P_t – no short-sale constraints

The optimization problem then becomes:

$$\begin{aligned} \max_{\xi} \quad u(C_t) &+ \beta \mathrm{E}_t[u(C_{t+1})] \quad \text{s.t.} \\ C_t &= e_t - P_t\xi \\ C_{t+1} &= e_{t+1} + X_{t+1}\xi \end{aligned}$$

F.O.C.:

$$P_t u'(C_t) = \mathrm{E}_t\left[\beta u'(C_{t+1}) X_{t+1}\right] \tag{2}$$

or equivalently

$$P_t = \mathrm{E}_t\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} X_{t+1}\right] \tag{3}$$

Comparing equation (3) with equation (1) we can define the stochastic discount factor m_{t+1} as

$$m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}. \tag{4}$$

- the current price is the discounted future payoff, taking uncertainty and investor's risk preferences into account
- m_{t+1} is sometimes also called marginal rate of substitution, since it determines at which rate the investor is willing to substitute consumption at time $t+1$ for consumption at time t
- m_{t+1} is also called a pricing kernel, a change of measure or state-price density since when you express the expectation as an integral the objective density (measure) can be considered to be transformed by m_{t+1}

Asset Specific Discount Factor

The basic pricing equation for asset i is

$$P_t^i = \mathrm{E}_t[m_{t+1} X_{t+1}^i],$$

is a generalization of

$$P_t^i = \frac{1}{R_t^i} \mathrm{E}_t[X_{t+1}^i], \tag{5}$$

We see though that m_{t+1} is not asset-specific. Since it is stochastic however, it covaries with X_{t+1}^i. This asset specific covariance generates asset-specific risk corrections.

Applicability

	Price P_t	Payoff X_{t+1}
Stock	P_t	$P_{t+1} + D_{t+1}$
Return	1	R_{t+1}
Excess return	0	$R^e_{t+1} = R^a_{t+1} - R^b_{t+1}$
One-period bond	P_t	1
Risk free rate	1	R^f
Option	C	$\max(S_T - K, 0)$

Real vs. Nominal Discount Factor

If prices and payoffs are in nominal terms we can transform them to real prices by

$$\frac{P_t}{\Pi_t} = \mathrm{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{X_{t+1}}{\Pi_{t+1}} \right],$$

This can be rewritten as

$$P_t = \mathrm{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{\Pi_t}{\Pi_{t+1}} X_{t+1} \right],$$

So $m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{\Pi_t}{\Pi_{t+1}}$ is a nominal discount factor with Π_t the price level at time t.

Selected Implications of the Basic Pricing Equation

Risk Free Rate

Investing one pound in the risk-free asset gives a payoff of R_t^f. Hence

$$1 = \mathrm{E}_t[m_{t+1} R_t^f] = \mathrm{E}_t[m_{t+1}] R_t^f.$$

It follows that

$$R_t^f = \frac{1}{\mathrm{E}_t[m_{t+1}]}. \tag{6}$$

Alternative interpretation: we **define** it to be like this; interpretation of a zero-beta asset.

Risk Corrections

Rewrite the BPE as

$$P_t = \mathrm{Cov}_t[m_{t+1}X_{t+1}] + \mathrm{E}_t[m_{t+1}]\mathrm{E}_t[X_{t+1}] = \frac{\mathrm{E}_t[X_{t+1}]}{R_t^f} + \mathrm{Cov}_t[m_{t+1}X_{t+1}].$$

From the consumption-based model we obtain

$$P_t = \frac{\mathrm{E}_t[X_{t+1}]}{R_t^f} + \underbrace{\frac{\mathrm{Cov}_t[\beta u'(C_{t+1}), X_{t+1}]}{u'(C_t)}}_{\text{correction for risk}}. \tag{7}$$

- X_{t+1} and C_{t+1} – positively correlated $\Rightarrow$ price is ...?
- X_{t+1} and C_{t+1} – negatively correlated $\Rightarrow$ price is ...?

Covariance, rather than variance determines the price. Intuition? Note that the investor cares about the variance of her *consumption*, not the variance of her assets. Consider buying an additional small amount ξ of the asset: The variance of consumption is then

$$\mathrm{Var}[C_{t+1} + \xi X_{t+1}] = \mathrm{Var}[C_{t+1}] + 2\xi \mathrm{Cov}[C_{t+1}, X_{t+1}] + \xi^2 \mathrm{Var}[X_{t+1}].$$

- Covariance – first order effect
- Variance – second order effect

Return Representation:

Invest 1 pound in asset i:

$$1 = \mathrm{E}_t[m_{t+1}R^i_{t+1}] = \mathrm{Cov}_t[m_{t+1}, R^i_{t+1}] + \mathrm{E}_t[m_{t+1}]\mathrm{E}_t[R^i_{t+1}].$$

Divide by $\mathrm{E}_t[m_{t+1}]$ and substitute the consumption-based model:

$$\mathrm{E}_t[R^i_{t+1}] - R^f_t = -\frac{\mathrm{Cov}_t[u'(C_{t+1}), R^i_{t+1}]}{\mathrm{E}_t[u'(C_{t+1})]}.$$

Interpretation: Three cases depending on the correlation between C_{t+1} and R^i_{t+1}

- No correlation (zero-beta asset)
- Positive correlation
- Negative correlation

Idiosyncratic Risk

The risk of the payoff of an asset can be decomposed into a systematic part and an idiosyncratic part (uncorrelated with the discount factor). No matter how volatile an asset is, if we have that $\mathrm{Cov}_t[m_{t+1}X_{t+1}] = 0$, then

$$P_t = \frac{\mathrm{E}_t[X_{t+1}]}{R_t^f}.$$

Using a regression approach we can separate the part of the payoff which is correlated with the discount factor m and a residual part, uncorrelated with m. Consider the regression

$$X_{t+1} = \beta_t m_{t+1} + \varepsilon_{t+1}.$$

We know from regression theory that it holds that

$$\beta_t = \frac{\mathrm{E}_t[m_{t+1}X_{t+1}]}{\mathrm{E}_t[m_{t+1}^2]}$$

and

$$\mathrm{E}_t[m_{t+1}\varepsilon_{t+1}] = 0,$$

We have decomposed X_{t+1} into

- $\beta_t m_{t+1}$ – perfectly correlated with m_{t+1}
- ε_{t+1} – uncorrelated with $m_{t+1} \Rightarrow$ zero-price

Consider the price $\tilde{P}_t$ of the payoff $\tilde{X}_{t+1} = \beta_t m_{t+1}$ and compare it to the price P_t of the payoff X_{t+1}.

$$\tilde{P}_t = \mathrm{E}_t[m_{t+1}\tilde{X}_{t+1}] = \mathrm{E}_t\left[m_{t+1}^2 \frac{\mathrm{E}_t[m_{t+1}X_{t+1}]}{\mathrm{E}_t[m_{t+1}^2]}\right] = \mathrm{E}_t[m_{t+1}X_{t+1}] = P_t.$$

Only systematic risk is priced, while idiosyncratic risk does not affect the price!

Beta Representation

Consider again

$$1 = \mathrm{E}_t[m_{t+1} R^i_{t+1}] = \mathrm{Cov}_t[m_{t+1}, R^i_{t+1}] + \mathrm{E}_t[m_{t+1}]\mathrm{E}_t[R^i_{t+1}].$$

Dividing by $\mathrm{E}_t[m_{t+1}]$ and using the equation for the risk free rate we obtain

$$\mathrm{E}_t[R^i_{t+1}] = R^f_t + \left(\frac{\mathrm{Cov}_t[m_{t+1}, R^i_{t+1}]}{\mathrm{V}_t[m_{t+1}]}\right)\left(-\frac{\mathrm{V}_t[m_{t+1}]}{\mathrm{E}_t[m_{t+1}]}\right).$$

This equation has the well known form

$$\mathrm{E}_t[R^i_{t+1}] = R^f_t + \beta^{i,m}_t \lambda^m_t,$$

with λ^m_t the price of risk.

Mean-Variance Frontier

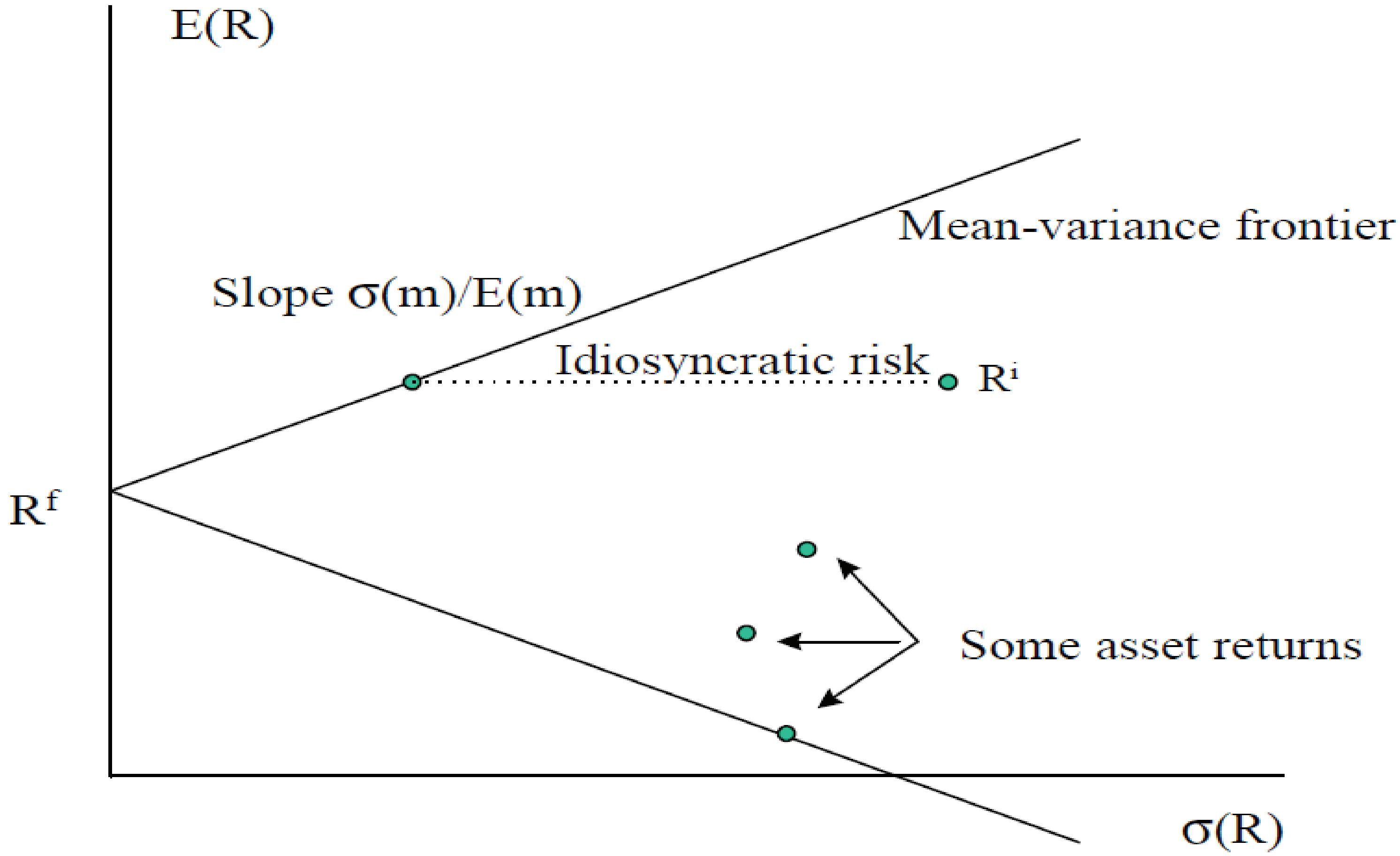
Start again from the BPE

$$\begin{aligned} 1 = \mathrm{E}_t[m_{t+1}R^i_{t+1}] &= \mathrm{E}_t[m_{t+1}]\mathrm{E}_t[R^i_{t+1}] + \mathrm{Corr}_t[m_{t+1}, R^i_{t+1}]\mathrm{Std}_t[m_{t+1}]\mathrm{Std}_t[R^i_{t+1}] \\ \mathrm{E}_t[R^i_{t+1}] &= R^f_t - \mathrm{Corr}_t[m_{t+1}, R^i_{t+1}]\frac{\mathrm{Std}_t[m_{t+1}]}{\mathrm{E}_t[m_{t+1}]}\mathrm{Std}_t[R^i_{t+1}]. \end{aligned} \tag{8}$$

It follows that

$$|\mathrm{E}_t[R^i_{t+1}] - R^f_t| \le \frac{\mathrm{Std}_t[m_{t+1}]}{\mathrm{E}_t[m_{t+1}]}\mathrm{Std}_t[R^i_{t+1}]. \tag{9}$$

Equation (8) is a straight line.


E(R)
Mean-variance frontier
Slope σ(m)/E(m)
Idiosyncratic risk
R^i
R^f
Some asset returns
σ(R)

The boundary of the mean-variance region is called the mean-variance frontier. All returns on the frontier are perfectly correlated with the discount factor, i.e. $|\mathrm{Corr}_t[m_{t+1}, R^{mv}_{t+1}]| = 1$ for all returns R^{mv} on the frontier. Returns which are perfectly negatively correlated with m lie on the upper part(the upward sloping line) of the frontier, while returns which are perfectly positively correlated with m (and hence perfectly negatively correlated with consumption) lie on the lower part of the frontier.

Since all returns on the frontier are perfectly correlated with the discount factor, we have that we can express

$$\begin{aligned} m_t &= a + bR_t^{mv} \\ R_t^{mv} &= c + dm_t \end{aligned}$$

Thus, any mean-variance efficient return carries all pricing information. Given a mean-variance efficient return and a risk-free rate we can infer the discount factor that prices all assets and vice versa.

We can use any mean-variance efficient return (except the risk-free) to construct a single-beta representation:

$$\mathrm{E}_t[R^i_{t+1}] = R^f_t + \beta^{i,mv}_t(\mathrm{E}_t[R^{mv}_{t+1}] - R^f_t).$$

A plot of expected returns against betas yields a straight line.

Since we can apply this representation for any return, including R^{mv}, which has a beta of 1 on itself, we can identify the risk premium as $\lambda = \mathrm{E}_t[R^{mv}_{t+1} - R^f_t]$

Note:

The basic pricing equation

$$P_t = \mathrm{E}_t[m_{t+1} X_{t+1}]$$

does **not** involve assumptions as

- Complete markets or the existence of a representative investor,
- Any particular distribution or dynamic properties of asset returns,
- Any particular utility functions (except that we require a monotone and concave utility function) or investment horizons,
- That the market has reached equilibrium or that individuals have bought all the securities they want to.

General Equilibrium:

Does consumption determine prices (returns) or vice versa?

Taking a different perspective the basic pricing equation can be written as

$$u'(C_t) = \mathrm{E}_t[\beta u'(C_{t+1})X_{t+1}/P_t].$$

In this form the equation can be thought of as determining today's consumption given the asset price and payoff.

In fact they are both endogenous and determined within a general equilibrium model depending on truly exogenous factors – current production technologies, current state of the markets.

– fixed technology, consumption adjusts in order for equilibrium to arise

– "endowment economy" – a situation in which asset prices adjust to consumption

– general equilibrium in which preferences (represented by indifference curve) and the production technology determine the equilibrium rate of return.

We can adopt one of the following approaches without using an equilibrium model:

- Form a statistical model of bond and stock returns, solve the optimal consumption-portfolio decision. Use the equilibrium consumption values to solve $P_t = \mathrm{E}_t[m_{t+1}X_{t+1}]$.
- Form a statistical model for the consumption process and calculate asset prices directly from the basic pricing equation.

Consumption Based Model (Applications):

The consumption-based model can be used to solve any problem in asset pricing. It can be applied to any security or uncertain cash flow. We only need to specify the utility, find numerical values for the parameters and a statistical model for the joint DGP of consumption and payoffs.

With power utility function, we have that $u'(C_t) = C_t^{-\gamma}$ and the discount factor is given by

$$m_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

Then if we apply the basic pricing equation for excess returns we obtain

$$0 = \mathrm{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^e_{t+1} \right]$$

Taking unconditional expectations and using the risk-free rate equation we have for the expected excess return that

$$\mathrm{E}[R^e_{t+1}] = -R^f_t \,\mathrm{Cov} \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}, R^e_{t+1} \right]$$

Given a value of γ, and data on consumption and returns, expected returns and the covariance on the right hand side can easily be estimated by their sample counterparts.

Further examples:

For an N- period nominal discount bond paying one dollar at $t+N$, the price is given by

$$P_t = \mathrm{E}_t \left[\beta^N \left(\frac{C_{t+N}}{C_t} \right)^{-\gamma} \frac{\Pi_t}{\Pi_{t+N}} 1 \right],$$

where Π is a price level indicator. The price of an European call option with maturity $t+T$ and a strike price K should be

$$P_t = \mathrm{E}_t \left[\beta^T \left(\frac{C_{t+T}}{C_t} \right)^{-\gamma} \max(S_{t+T} - K, 0) \right],$$

where S_{t+T} is the stock price at time $t+T$.

Unfortunately, this type of model turns out to provide poor predictions about actual expected returns. Possible reasons might be

- Misspecified utility function
- Unreliable consumption data
- Factor pricing models – proxy the marginal utility in terms of “factors”

$$m_{t+1} = a + b_1 f^1_{t+1} + b_2 f^2_{t+1} + \ldots$$

- Arbitrage pricing – Black-Scholes

Factor Pricing Models

Factor pricing models look for factors for which

$$\beta \frac{u'(C_{t+1})}{u'(C_t)} \approx a + b' f_{t+1}$$

is a sensible and economically plausible approximation.

For example, the arbitrage pricing theory (APT) and the Intertemporal CAPM (ICAPM) have such a form. In the APT, factors are determined by statistical methods. In the ICAPM the factors are “state variables” which serve to describe the possible future state of the world (or the part of it that matters to the particular investor).

The CAPM is also a factor pricing model with

$$m_{t+1} = a_t + b_t R^W_{t+1}.$$

R^W_{t+1} is usually proxied by a broad-based portfolio such as some weighted version of all NYSE stocks, the S&P 500, etc. The CAPM is best known in its expected return-beta representation

$$\mathrm{E}_t[R^i_{t+1}] = R^f_t + \beta^{i,R^W}_t \left(\mathrm{E}_t[R^W_{t+1}] - R^f_t\right).$$

Two features:

- Linear in the factor
- Defending R^W as the factor that proxies for consumption.

The CAPM can be derived from the consumption-based model under different sets of assumptions as follows

- Two-period quadratic utility
- Two periods, exponential utility and normally distributed returns
- Infinite horizon, quadratic utility and i.i.d. returns
- Log utility

Here we present the derivation in the first case with **Two-Period Quadratic Utility**.

With quadratic utility we have that

$$U(C_t, C_{t+1}) = -\frac{1}{2}(C^* - C_t)^2 - \frac{1}{2}\beta \mathrm{E}_t\left[(C^* - C_{t+1})^2\right].$$

This utility function is concave with a maximum at $C = C^*$.

The discount factor with this utility is thus given by

$$m_{t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)} = \beta\frac{C^* - C_{t+1}}{C^* - C_t}.$$

One result from assuming quadratic utility is hence linearity of the discount factor in consumption.

Assume investors are born in t with wealth W_t and earn no labor income. They can invest in N assets with prices P_t^i and payoffs X_{t+1}^i, or equivalently with returns R_{t+1}^i. Investors must choose their optimal consumption levels C_t and C_{t+1}, and portfolio weights w_i. The maximization problem is given by

$$\begin{aligned}
\max_{C_t, C_{t+1}, w_i} \quad & -\frac{1}{2}(C^* - C_t)^2 - \frac{1}{2}\beta \mathrm{E}_t\left[(C^* - C_{t+1})^2\right] \quad \text{subject to} \\
C_{t+1} &= W_{t+1} \\
W_{t+1} &= R_{t+1}^W (W_t - C_t) \\
R_{t+1}^W &= \sum_{i=1}^N w_i R_{t+1}^i \\
\sum_{i=1}^N w_i &= 1.
\end{aligned}$$

$$m_{t+1} = \beta \frac{C^* - R^W_{t+1}(W_t - C_t)}{C^* - C_t} = \frac{\beta C^*}{C^* - C_t} - \frac{\beta(W_t - C_t)}{C^* - C_t} R^W_{t+1}.$$

Hence we arrive at

$$m_{t+1} = a_t + b_t R^W_{t+1},$$

with

$$\begin{aligned} a_t &= \frac{\beta C^*}{C^* - C_t} \\ b_t &= -\frac{\beta(W_t - C_t)}{C^* - C_t}. \end{aligned}$$

Written in terms of am expected return-beta representation we have

$$\mathrm{E}_t[R^i_{t+1}] = R^f_t + \beta^{i,R^W}_t \left(\mathrm{E}_t[R^W_{t+1}] - R^f_t \right). \tag{10}$$

– conditional vs. unconditional CAPM

Conditional CAPM with MGARCH

Models that can handle conditional moments: GARCH class. We write Equation (10) shifting it one period back as

$$\begin{aligned} \mathrm{E}_{t-1}[R_t^i] &= R_{t-1}^f + \frac{\mathrm{Cov}_{t-1}[R_t^i, R_t^W]}{\mathrm{V}_{t-1}[R_t^W]} \left(\mathrm{E}_{t-1}[R_t^W] - R_{t-1}^f \right) \\ &= R_{t-1}^f + \lambda_t \mathrm{Cov}_{t-1}[R_t^i, R_t^W], \end{aligned} \tag{11}$$

where

$$\lambda_t = \frac{\mathrm{E}_{t-1}[R_t^W] - R_{t-1}^f}{\mathrm{V}_{t-1}[R_t^W]}.$$

Since $R_t^W = \sum_{i=1}^N w_{ti} R_t^i$ we can write the system of N equations of the type (11) in matrix form

$$\mu_t = R_t^f \iota + \lambda_t H_t w_t$$

where: $\mu_t = \mathrm{E}_{t-1}\left[(R_{1,t}, \ldots, R_{N,t})'\right]$, ι is an $N \times 1$ vector of ones, H_t is the conditional covariance matrix of the return vector R_t, and w_t is the vector of asset weights.

An econometric model compatible with this formulation is:

$$
\begin{aligned}
R_t - R_t^f \iota &= \underbrace{\mu\iota + \lambda_t H_t w_t}_{\text{cond. mean function}} + \varepsilon_t && (12) \\
\lambda_t &= \lambda_0 + \lambda_1 f(H_t) && (13) \\
\mathrm{E}_{t-1}\left[\varepsilon_t\right] &= 0 \\
\mathrm{Var}_{t-1}\left[\varepsilon_t\right] &= H_t \\
\varepsilon_t \quad iid & \sim \text{arbitrary distribution}
\end{aligned}
$$

with R_t^f as a risk free return and H_t – an MGARCH formulation.

- λ_t may be constant $(\lambda_1 = 0)$ or time dependent ($\lambda_1 \neq 0$) through a function of the conditional variance matrix.
- Having estimated μ, λ_t, and H_t we can easily compute the betas!

Application of Bollerslev, Engle, Wooldridge (1988)

$$R_t - R_t^f = \mu + \lambda H_t w_t + \varepsilon_t, \quad \varepsilon_t | \mathcal{F}_{t-1} \sim \mathrm{N}(0, H_t) \tag{14}$$

with

$$vech(H_t) = C + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_i vech(H_{t-j}), \quad \text{a VEC}(p,q) \text{ model}$$

To simplify: impose diagonality constraints on the matrices A_i and B_j and further assume that $p = q = 1$, thus (14) becomes

$$\begin{aligned} R_{it} - R_t^f &= \mu_i + \lambda \sum_j w_{jt} h_{ij,t} + \varepsilon_{it} \\ \varepsilon_t | \mathcal{F}_{t-1} &\sim \mathrm{N}(0, H_t) \\ h_{ij,t} &= c_{ij} + a_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij} h_{ij,t-1} \quad i, j = 1, \ldots, N. \end{aligned}$$

The market portfolio is assumed to be built by 3 asset groups

- 6-month T-bills
- 20-year T-bonds
- Stocks

GMM Estimation of the consumption based model, the CAPM and the Fama-French 3 factor model

Moments: 25 "Fama-French" portfolios sorted on the basis of size (market equity) and book/market ratio The three models can be represented by the discount factor

- CBM: $m_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$
- CAPM: $m_{t+1} = a_t + b_t R^W_{t+1}$
- FF: $m_{t+1} = a + b_1 f^1_{t+1} + b_2 f^2_{t+1} + b_3 f^3_{t+1}$

The three factors in the Fama-French model include the market portfolio, a small minus big portfolio (SMB) and a high minus low portfolio (HML).

Readings:

- Cochrane, J (2000): *Asset Pricing*, Princeton University Press, Princeton.
- Hansen L & K Singelton (1982): Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica*, Vol. 50, No. 5, pp. 1269-1286
- Bollerslev T, R. Engle R & M Wooldridge (1998): A Capital Asset Pricing Model with Time-Varying Covariances, *The Journal of Political Economy*, Vol. 96, No. 1, pp. 116-131